USEFUL FORMULAE

$$E(X)=\sum_{i=1}^{k} p_i x_i$$

$$Var(X)=E\left[(X-\mu_X)^2\right]=\sum_{i=1}^{k}(x_i-\mu_X)^2 p_i$$

$$\Pr(X=x)=\sum_{i=1}^{k}\Pr(X=x,Y=y_i)$$

$$\Pr(Y=y\mid X=x)=\frac{\Pr(X=x,Y=y)}{\Pr(X=x)}$$

$$E(Y\mid X=x)=\sum_{i=1}^{k} y_i \Pr(Y=y_i\mid X=x)$$

$$E(Y)=\sum_{i=1}^{m} E(Y\mid X=x_i)\Pr(X=x_i)$$

$$Var(Y\mid X=x)=\sum_{i=1}^{k}\left[y_i-E(Y\mid X=x)\right]^2 \Pr(Y=y_i\mid X=x)$$

$$E(a+bX+cY)=a+bE(X)+cE(Y)$$

$$Cov(X,Y)=\sum_{i=1}^{k}\sum_{j=1}^{m}(x_j-\mu_X)(y_i-\mu_Y)\Pr(X=x_j,Y=y_i)$$

$$Var(a+bY)=b^2Var(Y)$$

$$Corr(X,Y)=\rho_{XY}=\frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$$

$$Var(aX+bY)=a^2Var(X)+b^2Var(Y)+2abCov(X,Y)$$

$$E(Y^2)=Var(Y)+E(Y)^2$$

$$Cov(a+bX+cV,Y)=bCov(X,Y)+cCov(V,Y)$$

$$E(XY)=Cov(X,Y)+E(X)E(Y)$$

$$t=\frac{\bar{x}-\mu}{s/\sqrt{n}}$$

$$Z=\frac{X-\mu}{\sigma}$$

$$\bar{x}=\frac{1}{n}\sum_{i=1}^{n} x_i$$

$$s^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

$$s_{XY}=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})$$

$$r_{XY}=s_{XY}/s_X s_Y$$

$$\hat{Y}_i=\hat{\beta}_0+\hat{\beta}_1X_{1i}+\hat{\beta}_2X_{2i}+\cdots+\hat{\beta}_kX_{ki}$$

For the simple linear regression model $Y_i=\beta_0+\beta_1X_i+\varepsilon_i$ we have: $\hat{\beta}_1=\dfrac{\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y})}{\sum_{i=1}^{n}(X_i-\bar{X})^2}$ $\quad\hat{\beta}_0=\bar{Y}-\hat{\beta}_1\bar{X}$

For the even simpler regression model $Y_i=\beta X_i+\varepsilon_i$ we have: $\hat{\beta}=\dfrac{\sum_{i=1}^{n}(X_iY_i)}{\sum_{i=1}^{n}(X_i)^2}$